

On the Potential of the Irreducible Description of Complex Systems for the Modeling of the Global Financial System

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The recent financial crisis has sharply revealed that current understanding of the global financial system is more than limited. In the recovery plan the confidence in the underlying theory is crucial. To address the problem we propose the description of complex systems in terms of self-organization processes of prime integer relations. The description suggests to consider the global financial system through the hierarchical network built by the totality of the self-organization processes. To make the description operational we propose an integration principle and support it by computational experiments using a multi-agent system. Remarkably, based on integers and controlled by arithmetic only the description raises the possibility to develop an irreducible theory of complex systems. These may open a fundamentally new perspective for the modeling of the global financial system.

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I. INTRODUCTION

The recent financial crisis tests our approach to reality and defines us much by the reaction. It has sharply revealed that current understanding of the financial system, for a rather long time shaped and entangled by the globalization into a complex entity, is more than limited. We can not clearly see the way out of the crisis and, as uncertainty prevails, there is no visible ground to restore the confidence. At the same time a huge potential of goodwill has been accumulated recently for a change. In the recovery plan the confidence in the underlying theory is crucial.

However, since the stakes are extremely high, a certain aspect, usually not essential at all, has to be made very clear. Recommendations for the course of actions can be proposed only by using theoretical considerations derived from basic concepts for some good reasons assumed to be true. Yet, despite tremendous progress made in science and technology, so far there is no theory that originates from irreducible concepts. This actually means that in principle no theoretical considerations about reality can be completely trusted, because they are simply consequences of basic concepts.

In fact the situation is even more demanding. The global financial system will not be studied in a laboratory, where under different conditions experiments can be conducted as many times as needed. Moreover, as a regulatory system of the global economic development it can actually effect the world as a whole, the trial and error approach can not be acceptable. It is unlikely much time reserved to find the solution before it is too late.

II. SEARCH FOR AN IRREDUCIBLE THEORY OF COMPLEX SYSTEMS

A theory, sometimes known as a theory of everything [1] or irreducible [2], and now mainly of interest to meta-physical debates [3] and the unification of quantum mechanics and general relativity [4] may serve the occasion. However, such a theory appears elusive and highly unlikely to exist in principle. In fact, as experimental input is always limited, no matter how successful a theory can be, doubts will remain.

Yet we believe there is one possibility and it can not be missed: integers. It is a challenging task to develop a theory that could operate with statements like $1 + 2 = 3$ and produce results in agreement with observation. Because such statements are self-evident, the theory could obtain the irreducible status.

Remarkably, the description of complex systems in terms of self-organization processes of prime integer relations has been discovered [5]-[8] and showed promising results [9]-[14]. In particular, it turns out that self-evident integer relations can be organized into a hierarchical network, where an element of a level is formed from elements of the lower level so that all elements in the formation are necessary and sufficient. Moreover, such integer relations, we call them prime, can encode correlations between observables and when geometrized can describe the corresponding dynamics.

The description is realized through the unity of two equivalent forms, i.e., arithmetical and geometrical. In the arithmetical form a complex system is characterized by correlation structures built in accordance with self-organization processes of prime integer relations. In the geometrical form the processes become isomorphically represented by transformations of two-dimensional patterns determining the dynamics of the system. Based on the integers and controlled by arithmetic only the description raises the possibility to develop an irreducible

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theory of complex systems.

Importantly, the role of the description for the development of a new global financial system could be very special, because its recommendations would be derived from the concepts we can completely trust.

III. ON THE ROLE OF THE DESCRIPTION FOR THE MODELING OF THE GLOBAL FINANCIAL SYSTEM

Usually, complex systems are considered in space and time, where things are assumed to exist separately interacting by forces. Despite tremendous progress made through this approach, it still has severe limitations.

First, to understand a complex system in space and time all forces have to be identified. Only then it is possible to derive the equation of motion for the system and obtain the needed information. However, usually not only the precise character of the forces is unknown, but the forces themselves. This is especially relevant to the global financial system.

Second, since the forces are not unified in space and time, it is hard to establish general directions where forces might work together with certain consequences for complex systems. Without such directions it becomes difficult to search solutions, as possible options can not be properly ordered and appear quite similar. In particular, that may be one of the main reasons why NP-hard problems become a reality.

Moreover, it is not clear what arguments could be completely trusted to agree on the objective of the global financial system. Without this common understanding a long term coordination between different parties would be problematic.

In its turn our description suggests to consider things through a new stage rather than in space and time. This stage is the hierarchical network of prime integer relations - a structure built by the totality of the processes and existing through the mutual consistency of its parts. Therefore, in the description things are viewed as integrated parts of one whole.

Although such a holistic vision is not new [15], [16], yet in the description it becomes unique due to the irreducible mathematical structure [6], which has the potential to make this perspective operational. Significantly, the description promises to address the problems of complex systems mentioned above.

Namely, in our description all forces are managed behind the scene by a single "force" - arithmetic to serve the special purpose: to hold the parts of a system together and possibly drive its formation in one and the same direction to make the system more complex. Importantly, the forces do not exist separately, but through the self-organization processes of prime integer relations are unified and controlled to work coherently in the preservation and formation of complex systems.

Remarkably, a complex system can be seen as an inte-

grated part defined by certain processes in the hierarchical network of prime integer relations, as its spacetime dynamics is given by the geometry of the processes.

IV. INTEGRATION PRINCIPLE OF COMPLEX SYSTEMS

The description suggests us to formulate the following integration principle:

The objective of a complex system defined by self-organization processes of prime integer relations is to fit precisely into the processes the system is an integrated part of.

The principle appears as a universal objective of a complex system. In particular, from its perspective the optimization of a complex system is about fitting and preserving the position of the system in the corresponding processes.

The geometrical form of the description provides an important interpretation of the integration principle. In particular, the position of a system in the processes can be associated with a certain two-dimensional shape, which the geometrical pattern of the system has to take to satisfy the integration principle. Therefore, this suggests that in the realization of the integration principle it is important to compare the current geometrical pattern of the system with the one required for the system by the integration principle. Since the geometrical patterns are two-dimensional the difference between their areas can be useful to estimate the result.

Furthermore, the character of the processes may give an efficient way for the realization of the integration principle. As our processes work to make systems more complex they move, level by level, in one and the same direction. In the description the area of the geometrical pattern can be connected with the information about the system and thus its entropy. Remarkably, the area should monotonically increase with the complexity level. Indeed, with each consecutive level parts of a system combine and make the system more complex and thus more information is required to characterize the system.

As with each next level $l < k$ the geometrical pattern of a system could become closer to the geometrical pattern specified by the integration principle at level k , the performance of the system might increase but up to level k , where it attains the global optimum, and decrease after as with each consecutive level $l > k$ the geometrical pattern of the system would differ more from the required.

Therefore, as the area of the geometrical pattern of a system increases with the complexity level l , the performance of the system might behave as a concave function of the complexity with the global optimum attained at the level k specified by the integration principle.

The following experiments [9], [10] based on a multi-agent system provide the evidence supporting the integration principle.

V. COMPUTATIONAL EXPERIMENTS AND OPTIMALITY CONDITION

Traditionally, when the objective of a complex system can be formulated explicitly, optimization algorithms can be used to solve the problem. This provides an important context to test the integration principle by its realization in finding the global optimum. For this purpose computational experiments have been conducted by using a multi-agent system.

In particular, an optimization algorithm \mathcal{A} , as a complex system, of N computational agents minimizing the average distance in the travelling salesman problem (TSP) is developed [9].

Let agents start in the same city and choose the next city at random. Then at each step an agent visits the next city by using one of the two strategies: random or greedy. In the solution of a problem with n cities the state of the agents at step $j = 1, \dots, n-1$ can be described by a binary sequence $s_j = s_{1j} \dots s_{Nj}$, where $s_{ij} = +1$, if agent $i = 1, \dots, N$ uses the random strategy and $s_{ij} = -1$, if the agent i uses the greedy strategy.

The dynamics of the system is realized by the strategies the agents choose step by step and can be encoded by an $N \times (n-1)$ binary strategy matrix

$$S = \{s_{ij}, i = 1, \dots, N, j = 1, \dots, n-1\}.$$

Remarkably, the matrix can specify N stocks of a financial market instead of N agents in the TSP problem.

The complexity of the algorithm \mathcal{A} is tried to be changed monotonically by forcing the system to make the transition from regular behavior to chaos by period-doubling. To control the system in this transition a parameter $v, 0 \leq v \leq 1$ is introduced. It specifies a threshold point dividing the interval of current distances passed by the agents into two parts, i.e., successful and unsuccessful. This information is needed for an optimal if-then rule [17] each agent uses to choose the next strategy. The rule relies on the Prouhet-Thue-Morse (PTM) sequence

$$+1 - 1 - 1 + 1 - 1 + 1 + 1 - 1 \dots$$

and has the following description:

1. if the last strategy is successful, continue with the same strategy.
2. if the last strategy is unsuccessful, consult PTM generator which strategy to use next.

Remarkably, it is found that for each problem p tested from a class \mathcal{P} the performance of the algorithm \mathcal{A} behaves as a concave function of the control parameter with the global maximum at a value $v^*(p)$. The global maximums $\{v^*(p), p \in \mathcal{P}\}$ are used to probe whether the complexities of the algorithm \mathcal{A} and the problem are related.

By using the strategy matrices

$$\{S(v^*(p)), p \in \mathcal{P}\}$$

corresponding to the global maximums $\{v^*(p), p \in \mathcal{P}\}$ we characterize the geometrical pattern of the algorithm \mathcal{A} and its complexity. In particular, the complexity $C(\mathcal{A}(p))$ of the algorithm \mathcal{A} is approximated by the quadratic trace

$$C(\mathcal{A}(p)) = \frac{1}{N^2} \text{tr}(V^2(v^*(p))) = \frac{1}{N^2} \sum_{i=1}^N \lambda_i^2$$

of the variance-covariance matrix $V(v^*(p))$ obtained from the strategy matrix $S(v^*(p))$, where $\lambda_i, i = 1, \dots, N$ are the eigenvalues of $V(v^*(p))$.

The complexity $C(p)$ of the problem p is approximated by the quadratic trace

$$C(p) = \frac{1}{n^2} \text{tr}(M^2(p)) = \frac{1}{n^2} \sum_{i=1}^n (\lambda'_i)^2$$

of the normalized distance matrix

$$M(p) = \{d_{ij}/d_{\max}, i, j = 1, \dots, n\},$$

where $\lambda'_i, i = 1, \dots, n$ are the eigenvalues of $M(p)$, d_{ij} is the distance between cities i and j and d_{\max} is the maximum of the distances.

To reveal a possible optimality condition the points with the coordinates

$$\{x = C(p), y = C(\mathcal{A}(p)), p \in \mathcal{P}\}$$

are considered. The result indicates a linear relationship between complexities and suggests the following optimality condition of the algorithm [10].

If the algorithm \mathcal{A} demonstrates the optimal performance for a problem p , then the complexity $C(\mathcal{A}(p))$ of the algorithm \mathcal{A} is in the linear relationship

$$C(\mathcal{A}(p)) = 0.67C(p) + 0.33 \quad (1)$$

with the complexity $C(p)$ of the problem p .

According to the optimality condition if the optimal performance takes place, then in terms of the complexity the dynamics of the algorithm \mathcal{A} is in a certain relation with the structure of the distance network.

Significantly, the optimality condition is a practical tool. Indeed, for a given problem p by using the distance matrix we can calculate the complexity $C(p)$ of the problem p and from (1) find the complexity $C(\mathcal{A}(p))$ of the algorithm \mathcal{A} . Then to obtain the optimal performance of the algorithm \mathcal{A} for the problem p we need only to adjust the control parameter for the algorithm \mathcal{A} to work with the complexity $C(\mathcal{A}(p))$.

Since the geometrical pattern defines the complexity in our description, the optimality condition may be interpreted as a realization of the integration principle. In particular, when the algorithm \mathcal{A} shows the optimal performance for a problem p , there are reasons to suggest that the condition (1) reads: the geometrical pattern of

the algorithm fits exactly into the geometrical pattern of the problem. The constants in (1) are to reconcile different units used in measuring of the geometrical patterns.

Importantly, due to the concavity of the performance of the algorithm and polynomial computational complexity of its operations, in the conducted experiments the NP-hardness of the TSP problem seems to disappear. This raises the possibility that in our description NP-hard problems can be avoided [13].

VI. CONCLUSIONS

In the paper we have suggested to consider the global financial system through a new stage rather than in space

and time. This new stage is the hierarchical network - the structure built by the totality of the self-organization processes of prime integer relations and existing through the mutual consistency of its parts. As a result, the global financial system appears not as a separate entity interacting by forces, but as an integrated part of one whole, where it finds the meaning and purpose.

To make the description operational we have suggested an integration principle of complex systems and presented computational evidence supporting it.

The results raise the possibility to develop an irreducible theory that for the first time with full confidence in its foundation could underly the modeling a new global financial system and open a fundamentally new perspective to resolve the global financial crisis once and for all.

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